M00/510/H(1)M

INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
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# MARKSCHEME

## May 2000

## MATHEMATICS

## Higher Level

## Paper 1

12 pages

# Paper 1 Markscheme

## Instructions to Examiners

**1 Method of Marking**

(a) All marking must be done using a **red** pen.

(b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.

(c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

**2 Abbreviations**

The markscheme may make use of the following abbreviations:

***M*** Marks awarded for **Method**

***A*** Marks awarded for an **Answer** or for **Accuracy**

***C*** Marks awarded for **Correct** answers (irrespective of working shown)

***R*** Marks awarded for clear **Reasoning**

**3 Follow Through (ft) Marks**

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise the error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii) award ***M*** marks for a correct method and ***A*(ft)** marks if the subsequent working contains no further errors.

**Follow through** procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through** procedure.

| Markscheme | | Candidate's Script | Marking | |
|---|---|---|---|---|
| $ 600 × 1.02 | ***M1*** | Amount earned = $ 600 × 1.02 | 8 | ***M1*** |
| = $ 612 | ***A1*** | = $602 | × | ***A0*** |
| $ (306 × 1.02) + (306 × 1.04) | ***M1*** | Amount = 301 × 1.02 + 301 × 1.04 | 8 | ***M1*** |
| = $ 630.36 | ***A1*** | = $ 620.06 | 8 | ***A1*****(ft)** |

**Note that** the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by '**(d)**' (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

**4 Using the Markscheme**

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Where alternative methods are included, they often refer to graphic dispaly calculator solutions, and they are indicated by **OR** *e.g.*

Mean = 59 ***(G2)***

**OR**

Mean = 7820/134 ***(M1)***
= 59 ***(A1)***

Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by '**(d)**' (to indicate that the marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$ , $\bar{u}$ , $\underline{u}$ ; $\tan^{-1} x$ for arctan *x*.

## 5 Accuracy of Answers

(a) In the case when the accuracy of the answer is **specified in the question** (for example: "find the size of angle *A* to the nearest degree") the maximum mark is awarded **only if** the correct answer is given to the accuracy required.

(b) When the accuracy is **not** specified in the question, then the general rule applies:

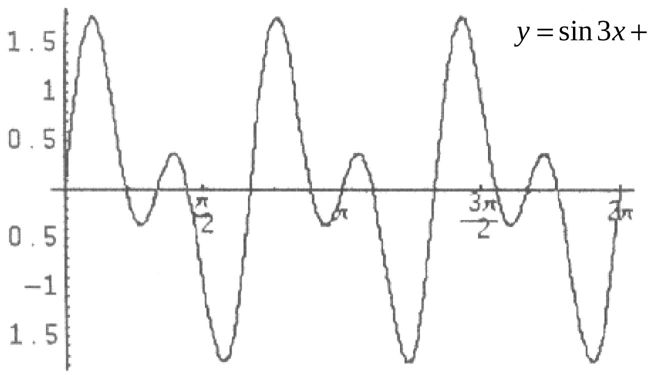
*Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.*

In this case, the candidate is **penalised once only IN THE PAPER** for giving a correct answer to the wrong degree of accuracy. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy maximum marks are **not** awarded, but on **all subsequent occasions** when correct answers are given to the wrong degree of accuracy then maximum marks **are** awarded.

## 6. Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. In general, written evidence would probably involve sketching a graph that is used to provide solutions. Where possible, examples will be provided to guide examiners in awarding these method marks.

**1.** (a)




$y = \sin 3x + \sin 6x$

***(A2)*** ***(C2)***

(b) Period $= \dfrac{2\pi}{3}$ ***(A1)*** ***(C1)***

***[3 marks]***

**2.** (a) $(\boldsymbol{T_1 T_2})^{-1} = \left(\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1}$ ***(M1)***

$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ***(A1)*** ***(C2)***

(b) This matrix represents a reflection in the *x*-axis. ***(A1)*** ***(C1)***

***[3 marks]***

**3.** $\left(\dfrac{z_1}{z_2}\right)^3 = \left(\dfrac{a^3\left(\cos\dfrac{3\pi}{4} + \mathrm{i}\sin\dfrac{3\pi}{4}\right)}{b^3(\cos\pi + \mathrm{i}\sin\pi)}\right)$ ***(M1)***

$= \dfrac{a^3}{b^3}\left(\cos\left(-\dfrac{\pi}{4}\right) + \mathrm{i}\sin\left(-\dfrac{\pi}{4}\right)\right)$ ***(A1)***

$= \left(\dfrac{\sqrt{2}}{2}\dfrac{a^3}{b^3}\right) - \left(\dfrac{\sqrt{2}}{2}\dfrac{a^3}{b^3}\right)\mathrm{i}$ or $\left(\dfrac{a^3}{\sqrt{2}b^3}\right) - \left(\dfrac{a^3}{\sqrt{2}b^3}\right)\mathrm{i}$ ***(A1)*** ***(C3)***

**OR**

$x = \dfrac{\sqrt{2}a^3}{2b^3},\ y = \dfrac{-\sqrt{2}a^3}{2b^3},$ or $x = \dfrac{a^3}{\sqrt{2}b^3},\ y = \dfrac{-a^3}{\sqrt{2}b^3}$ ***(A1)*** ***(C3)***

***[3 marks]***

**4.** (a) The sample standard deviation $s_n = 21.4$ hours. *(A2)* *(C2)*

(b) The unbiased estimate of the population standard deviation $s_{n-1} = 21.6$ hours. *(A1)* *(C1)*

***[3 marks]***

**5.** The required term is $\binom{10}{7} 2^{10-7} 3^7$ *(M2)*

$= 2\,099\,520$ *(A1)* *(C3)*

***[3 marks]***

**6.** Using Gaussian elimination, with the augmented matrix gives

$$\begin{pmatrix} 2 & -1 & -9 & 7 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & -3 & k \end{pmatrix}$$

$$\begin{matrix} \\ 2r_2 - r_1 \\ r_3 - r_1 \end{matrix} \begin{pmatrix} 2 & -1 & -9 & 7 \\ 0 & 5 & 15 & -5 \\ 0 & 2 & 6 & k-7 \end{pmatrix}$$ *(M1)*

$$\begin{matrix} \\ \\ 5r_3 - 2r_2 \end{matrix} \begin{pmatrix} 2 & -1 & -9 & 7 \\ 0 & 5 & 15 & -5 \\ 0 & 0 & 0 & 5k-25 \end{pmatrix}$$ *(M1)*

An infinite number of solutions exist only if $5k - 25 = 0$

$\Rightarrow k = 5$. *(A1)* *(C3)*

***[3 marks]***

**7.** $\sum_{\text{all } x} \mathrm{P}(X = x) = 1$

Therefore, $\frac{1}{5} + \frac{2}{5} + \frac{1}{10} + x = 1$

Therefore, $x = \frac{3}{10}$ *(A1)*

$\mathrm{P}(\text{scoring six after two rolls}) = \left(\frac{1}{10} \times \frac{1}{10}\right) + 2 \times \left(\frac{2}{5} \times \frac{3}{10}\right)$ *(M1)*

$= \frac{1}{4}$ *(A1)* *(C3)*

***[3 marks]***

**8.** A vector that is normal to the plane is given by the vector product $\boldsymbol{d}_1 \times \boldsymbol{d}_2$ where $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ are the direction vectors of the lines $L_1$ and $L_2$ respectively.

$$\boldsymbol{d}_1 \times \boldsymbol{d}_2 = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 1 & -2 \\ 0 & 1 & 3 \end{vmatrix}$$ ***(M2)***

$= 5\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$ (or any multiple) ***(A1)*** ***(C3)***

***[3 marks]***

**9.** $u_n = S_n - S_{n-1}$ ***(M1)***

$= \left[3n^2 - 2n\right] - \left[3(n-1)^2 - 2(n-1)\right]$ ***(A1)***

$= 6n - 5$ ***(A1)*** ***(C3)***

**OR**

$u_1 + u_n = 6n - 4$ ***(M1)(A1)***

$u_1 = 1$

$\Rightarrow u_n = 6n - 5$ ***(A1)*** ***(C3)***

***[3 marks]***

**10.** The direction vector, $\boldsymbol{i} + 2\boldsymbol{j} + l\boldsymbol{k}$, for the line, is perpendicular to $6\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$, the normal of the plane.

Therefore, $(\boldsymbol{i} + 2\boldsymbol{j} + l\boldsymbol{k}) \cdot (6\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}) = 0$ ***(M1)***

Therefore, $6 - 4 + l = 0$ ***(M1)***

$l = -2$ ***(A1)*** ***(C3)***

**OR**

$x = t + 1,\ y = 2t - 1,\ z = lt + 3$ ***(A1)***

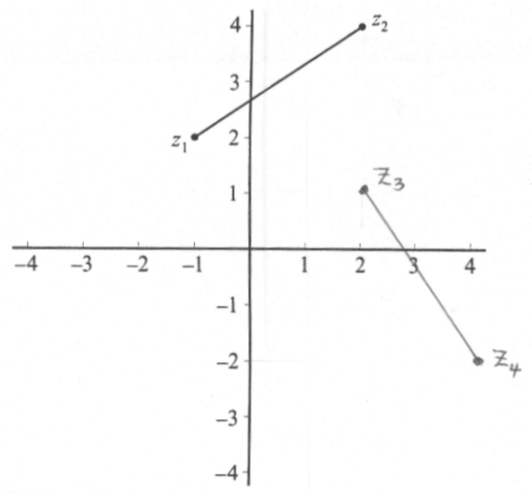
$6t + 6 - 4t + 2 + lt + 3 = 11$ ***(M1)***

$2t + lt = 0$

$l = -2$ ***(A1)***

***[3 marks]***

**11.** (a) $z_3 = -\mathrm{i}(-1+2\mathrm{i}) = \mathrm{i}+2$ and $z_4 = -\mathrm{i}(2+4\mathrm{i}) = -2\mathrm{i}+4$.




*(A2)* *(C2)*

| **Note:** Award ***(A1)*** for plotting $z_3$ and $z_4$ correctly, ***(A1)*** for drawing the line segment $[z_3z_4]$. |
|---|

(b) The transformation that maps the line segment $[z_1z_2]$ onto the line segment $[z_3z_4]$ is a rotation of $90^\circ$ clockwise (or $-90^\circ$) about the origin. *(A1)* *(C1)*

**OR**

Transformation matrix $= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ *(A1)* *(C1)*

***[3 marks]***

**12.** Since $X$ is a random variable, $\sum_{\text{all } x} \mathrm{P}(X=x) = 1$

Therefore, $k + \frac{2}{3}k + \left(\frac{2}{3}\right)^2 k + \left(\frac{2}{3}\right)^3 k + \ldots\ldots = 1$ *(M1)*

$$k\left(\frac{1}{1-\frac{2}{3}}\right) = 1$$ *(M1)*

$$k = \frac{1}{3}$$ *(A1)* *(C3)*

***[3 marks]***

**13.** Total distance $= k\int_0^a \mathrm{e}^{-t/2}\,\mathrm{d}t$ *(M1)*

$$= -2k\left[\mathrm{e}^{-t/2}\right]_0^a$$ *(M1)*

$= -2k(\mathrm{e}^{-a/2}-1)$ metres (or equivalent *e.g.* $2k(1-\mathrm{e}^{-a/2})$) *(A1)* *(C3)*

| **Note:** Award ***(C2)*** if $k$ is omitted in the final answer. |
|---|

***[3 marks]***

**14.** The number of different ways six people can sit around a circular table is $5! = 120$. ***(M1)***
The number of different ways these six people can sit around a circular table with Mr Black and Mrs White together is $4! \times 2 = 48$. ***(A1)***

Therefore, the number of different ways these six people can sit around a circular table with Mr Black and Mrs White **NOT** together is
$120 - 48 = 72$. ***(A1)*** ***(C3)***

**OR**

If Mr Black is seated, then Mrs White has 3 choices. ***(M1)***

4! is the number of different ways the remaining 4 people can sit around a circular table. ***(M1)***

Therefore the number of different ways these six people can sit around a circular table with Mr Black and Mrs White not together is $3 \times 4! = 72$. ***(A1)*** ***(C3)***

***[3 marks]***

**15.** Let *s* be the distance from the origin to a point on the line, then

$$s^2 = (1-\lambda)^2 + (2-3\lambda)^2 + 4$$
$$= 10\lambda^2 - 14\lambda + 9$$

$$\frac{\mathrm{d}(s^2)}{\mathrm{d}\lambda} = 20\lambda - 14$$ ***(M1)***

For minimum put $\frac{\mathrm{d}(s^2)}{\mathrm{d}\lambda} = 0, \Rightarrow \lambda = \frac{7}{10}$ ***(A1)***

The point is $\left(\frac{3}{10}, \frac{-1}{10}, 2\right)$. ***(A1)*** ***(C3)***

| |
|---|
| **Note:** At this point $\frac{\mathrm{d}^2(s^2)}{\mathrm{d}\lambda^2} > 0$, but this is not required |

**OR**

The position vector for the point nearest to the origin is perpendicular to the direction of the line. At that point:

$$\begin{pmatrix} 1-\lambda \\ 2-3\lambda \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} = 0$$ ***(M1)***

Therefore, $10\lambda - 7 = 0$

Therefore, $\lambda = \frac{7}{10}$ ***(A1)***

Therefore, the point is $\left(\frac{3}{10}, \frac{-1}{10}, 2\right)$. ***(A1)*** ***(C3)***

***[3 marks]***

**16.** Given $\dfrac{8x^3}{y} = 3$ (1)

$y = \sin x$

$xy - y = x^2 + \dfrac{9}{4}$ (2)

$y = \dfrac{8x^3}{3}$ ***(M1)***

Substituting into (2) gives

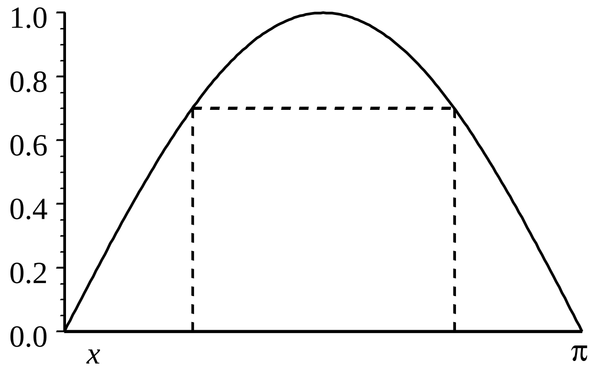
$32x^4 - 32x^3 - 12x^2 - 27 = 0$ (or equivalent) ***(A1)***

$\Rightarrow x = \dfrac{3}{2},\ y = 9$ ***(A1)*** ***(C3)***

**Notes:** Award final ***(A1)*** only if **both** values are correct.
If no working is shown, and only 1 answer is correct, award ***(C1)***.
Some candidates may be using calculators that cannot find these exact answers i.e. 3/2 and 9. Award marks as appropriate, where answers seem incorrect. Candidates should sketch graphs as part of their answers, and this should help identify why answers may be incorrect.
GDC example: finding solutions from a graph.

***[3 marks]***

**17.**




(a) Area $= (\pi - 2x)\sin x$. ***(M1)(A1)*** ***(C2)***

(b) Maximum Area $= 1.12$ units$^2$ ***(A1)*** ***(C1)***

***[3 marks]***

**18.** If $\displaystyle\int_a^{a^2} \frac{1}{1+x^2}\,dx = 0.22$

Then $\left[\arctan x\right]_a^{a^2} = 0.22$ ***(M1)***

$\arctan a^2 - \arctan a - 0.22 = 0$ ***(A1)***

$a = 2.04$ or $a = 2.62$ ***(A1)*** ***(C3)***

**Notes:** Award final ***(A1)*** only if both correct answers are shown.
If no working is shown and only one answers is correct, award ***(C1)***.
GDC example: finding solutions from a graph.

***[3 marks]***

**19.** (a) Given $f(x) = e^{\sin x}$

Then $f'(x) = \cos x \times e^{\sin x}$ ***(A1)*** ***(C1)***

(b) $f''(x) = \cos^2 x \times e^{\sin x} - \sin x \times e^{\sin x}$

$= e^{\sin x}(\cos^2 x - \sin x)$ ***(M1)***

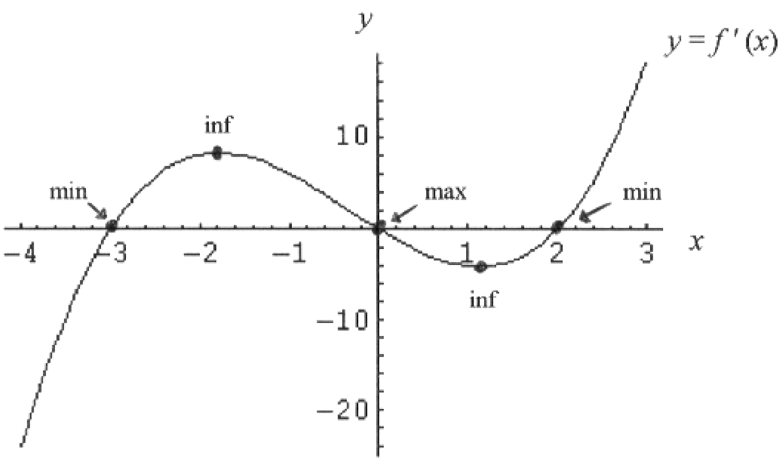
For the point of inflexion, put $f''(x) = 0$

$\Rightarrow \quad e^{\sin x}(\cos^2 x - \sin x) = 0$ (or equivalent) ***(A1)*** ***(C2)***

**Note:** Award ***(C1)*** if the candidate only writes $f''(x) = 0$.

***[3 marks]***

**20.**




(a) Minimum points ***(A1)*** ***(C1)***
(b) Maximum point ***(A1)*** ***(C1)***
(c) Points of inflexion ***(A1)*** ***(C1)***

**Note:** There is no scale on the question paper. For examiner reference the scale has been added here and the numerical answers are minima at $x = -3$ and 2, maximum at $x = 0$ and points of inflexion at $x = -1.79$ and 1.12.

***[3 marks]***